B.Tech Degree I & II Semester Examination in Marine Engineering June 2012

MRE 102 ENGINEERING MATHEMATICS II

Time : 3 Hours　　　　　　　　　　　　　　　　　　　　Maximum Marks : 100

I. (a) Find the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$. (5)

(b) Show that the following equations are consistent $3x+3y+2z=1,\ x+2y=4, 10y+3z=-2,\ 2x-3y-z=5$. Hence solve it. (10)

(c) Obtain the eigen values of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ 5)

OR

II. (a) State and prove the necessary condition for a function to be analytic. (7)

(b) Evaluate $\int_c \frac{e^{-z}dz}{z+1}$ where c is the circle $|z|=2$. (6)

(c) Expand $\frac{1}{z^2-3z+2}$ in the region (a) $|z|<1$ (7)

(b) $|z|>2$

(c) $1<|z|<2$

III. (a) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2}+y=\tan x$. (8)

(b) Solve (i) $xy\left(1+xy^2\right)\frac{dy}{dx}=1$. (6)

(ii) $xy\frac{dy}{dx}=1+x+y+xy$. (6)

OR

IV. (a) Solve (i) $\frac{d^2y}{dx^2}+a^2y=\tan ax$ (6)

(ii) $\frac{d^2y}{dx^2}-4y=x\sin hx$. (6)

(b) Solve the simultaneous equations (8)
$\frac{dx}{dt}+y=\sin t;\ \frac{dy}{dt}+x=Cot\,t$.

(P.T.O)

V. (a) Find the Fourier series to represent $f(x) = x^2 - 2; -2 \le x \le 2$. (10)

(b) Express $f(x) = \begin{cases} 1 \text{ for } 0 \le x \le \pi \\ 0 \text{ for } x > \pi \end{cases}$ as a Fourier sine integral and hence evaluate (10)

$\int_0^\infty \frac{1 - cos\pi\lambda}{\lambda} \sin x\lambda . d\lambda$

OR

VI. (a) Find the half range cosine series for $\sin x$ in $0 < x < \pi$. (10)

(b) Prove that $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma m+n}$. (10)

VII. (a) Find the Laplace transform of (i) $te^{-4t}\sin 3t$ (ii) $\frac{1-e^t}{t}$. (8)

(b) Find the inverse Laplace transform of (i) $\frac{S+8}{S^2+4S+5}$ (ii) $\frac{1}{S(S^2+4)}$. (12)

OR

VIII. (a) Solve by method of transforms $y'' - 3y' + 2y = 4t + e^{3t}$; $y(0) = 1$ and $y'(0) = -1$. (10)

(b) Derive the Laplace transform of periodic function. (10)

IX. (a) State Baye's Theorem. There are two bags – one of which contains 2 white and 4 black balls, other contains 4 white and 3 black balls. A ball is drawn from one of the bags and it turns out to be white. What is the probability that it comes from the first bag? (10)

(b) Find the mean and variance of the r.v. with pdf $f(x) = kx(1-x); 0 < x < 1$. (10)

OR

X. (a) Derive the mean and variance of binomial distribution. (10)

(b) If X is a Poisson variate such that $P[X=2] = a\hat{P}[X=4] + 90P[X=6]$. Find the variance. (5)

(c) If the rv X follows normal distribution with mean 25 and S.D 5, find $P[30 < x < 35]$. (5)

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